

# RF Topologies

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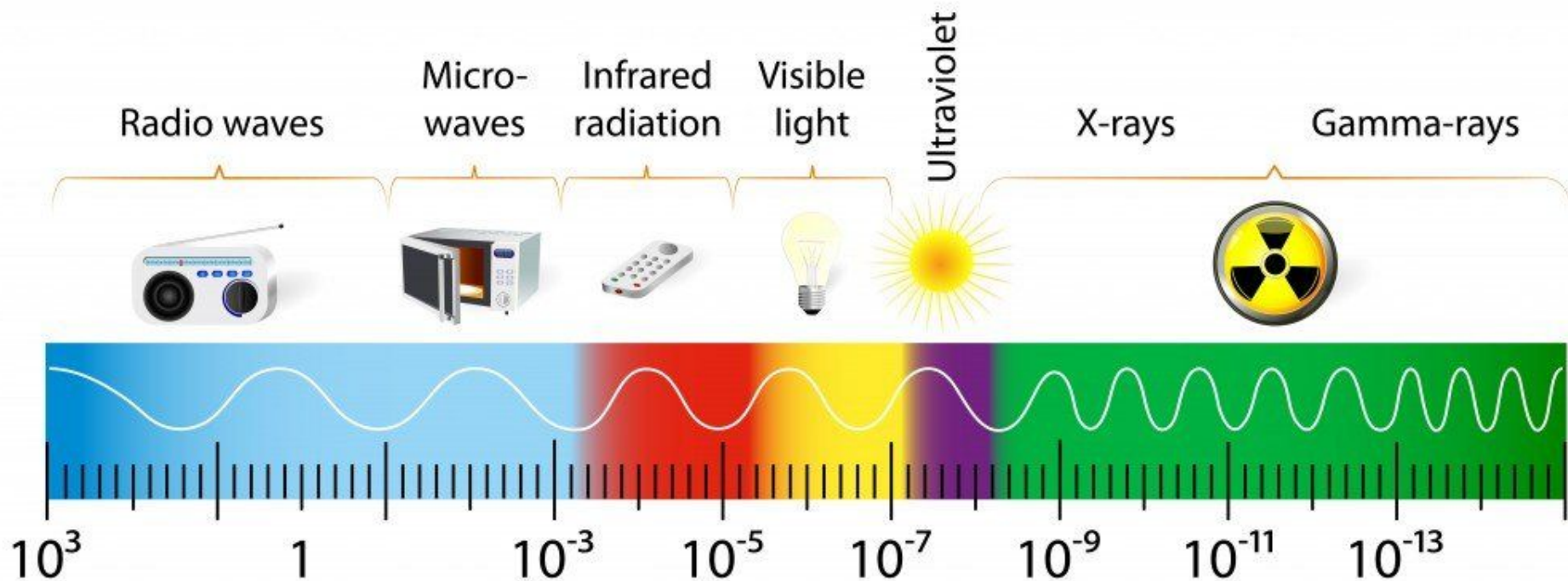
June 2018

# Goals

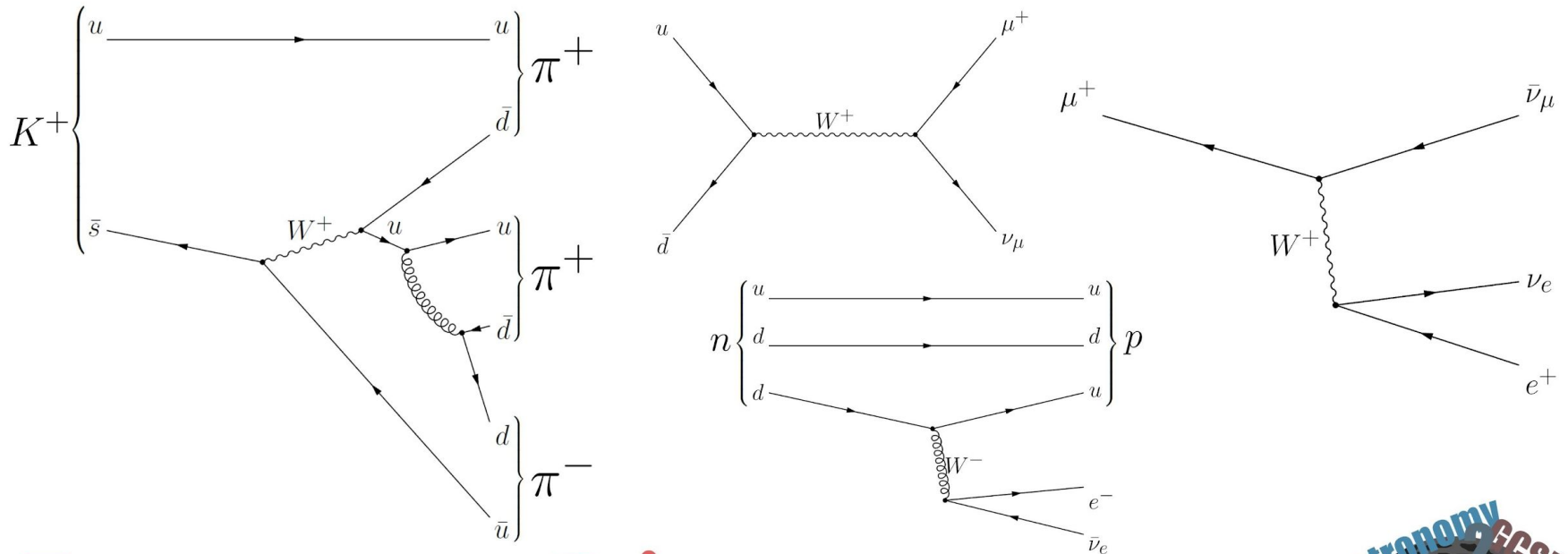
- Apply generalized topology diagrams towards standardizing WiFi test rig designs
- Better emulate over the air (OTA) scenarios when using “conducted equipment”
- Frequency band is 2.4-6 GHz
- Use “low cost” conducted equipment sold by outside vendors
- Automate with python

# Theory

# THE ELECTROMAGNETIC SPECTRUM



# An introduction to:

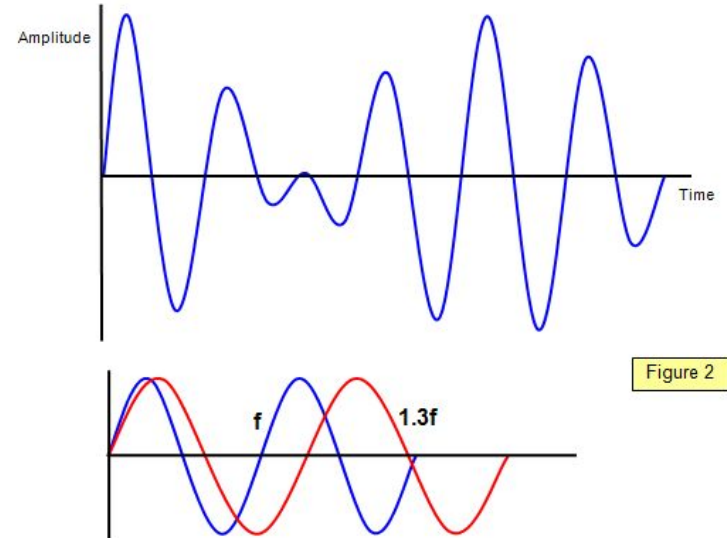
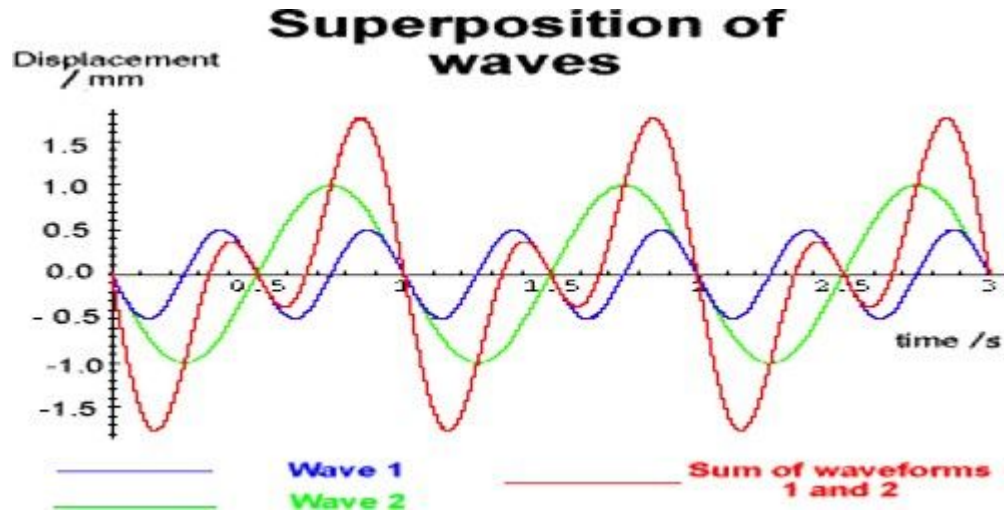


# Feynman Diagrams

astronomy2GCSE

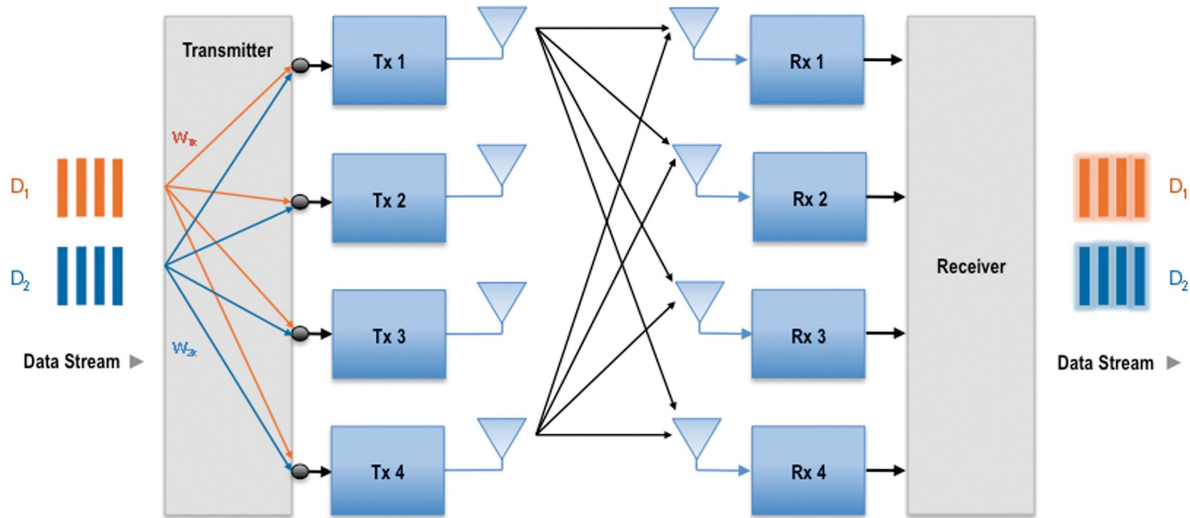


# Superposition



# Mathematical Models

# A Basic MiMo Model



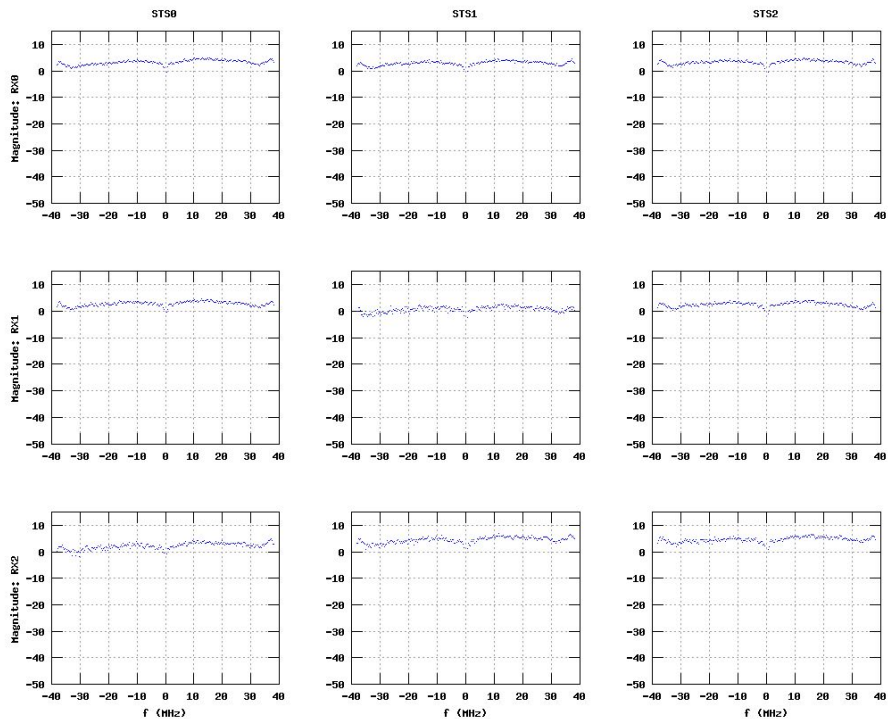


# Transfer Matrix

$$\begin{bmatrix} R1 \\ R2 \\ R3 \\ R4 \end{bmatrix} = \begin{bmatrix} D1 & 0 & 0 & 0 \\ 0 & D2 & 0 & 0 \\ 0 & 0 & D3 & 0 \\ 0 & 0 & 0 & D4 \end{bmatrix} \times \begin{bmatrix} h11 & h12 & h13 & h14 \\ h21 & h22 & h23 & h24 \\ h31 & h32 & h33 & h34 \\ h41 & h34 & h43 & h44 \end{bmatrix} \times \begin{bmatrix} D1' & 0 & 0 & 0 \\ 0 & D2' & 0 & 0 \\ 0 & 0 & D3' & 0 \\ 0 & 0 & 0 & D4' \end{bmatrix} \times \begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \end{bmatrix}$$

- S-Matrix is the transmitted signal (or energy) per antenna
- R-Matrix is the received signal
- D-Matrices are the cables and attenuators between the antenna and the H-Matrix (one for source and one for sink)
- H-Matrix is the MiMO “channel” (signal mixing, e.g butler matrix, splitter/combiner, power divider, MuMimo midbox)

# Phychanest



# Hermitian

## Diagonalizing Hermitian Matrices

- The eigenvalues of a **Hermitian Matrix** are always real
- For a Hermitian matrix the eigenvectors corresponding to two different eigenvalues are orthogonal
- A matrix has real eigenvalues and can be diagonalized by a unitary similarity transformation if and only if it is Hermitian
- A matrix has real eigenvalues and can be diagonalized by an orthogonal similarity transformation if and only if it is symmetric

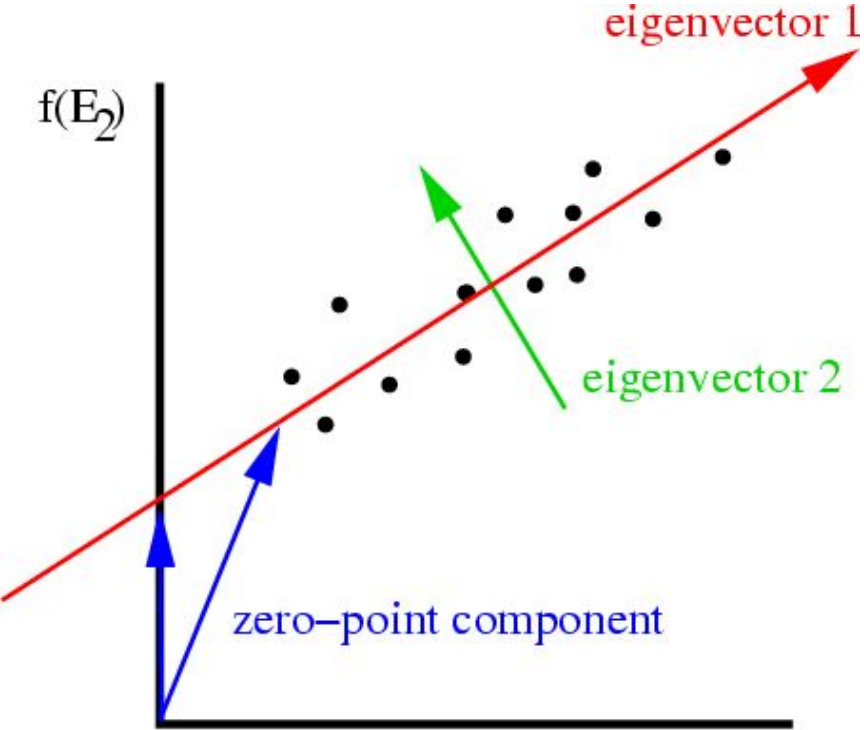
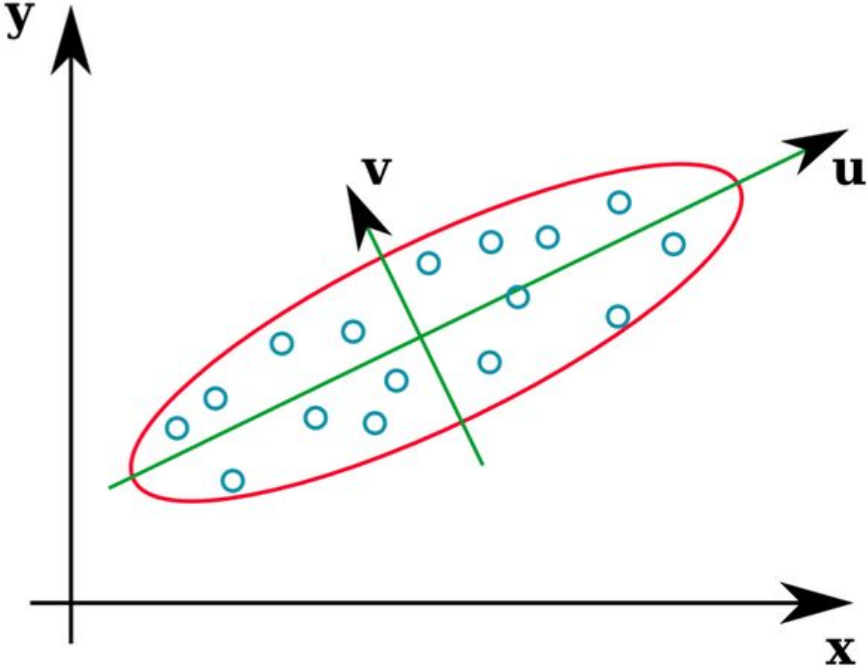
# “Eigens”

The diagram shows the equation  $\mathbf{Ax} = \lambda\mathbf{x}$  in the center. A callout box at the top points to the vector  $\mathbf{x}$  and is labeled "Eigenvector of Matrix A". A callout box at the bottom points to the scalar  $\lambda$  and is labeled "Eigenvalue of Matrix A".

Eigen 'in Germany' means 'Characteristics' in English. So you may guess, 'Eigen vector' would be a special vector that represents a specific characteristics of a Matrix (a Square Matrix) and 'Eigen value' would be a special value that represents a specific characteristics of a Matrix (a Square Matrix)

If you think of a Matrix as a geometric transformer, the Matrix usually perform two types of transformational action. One is 'scaling(extend/shrink)' and the other one is 'rotation'.

# Eigenvalues, Eigenvectors



# Graph Theory

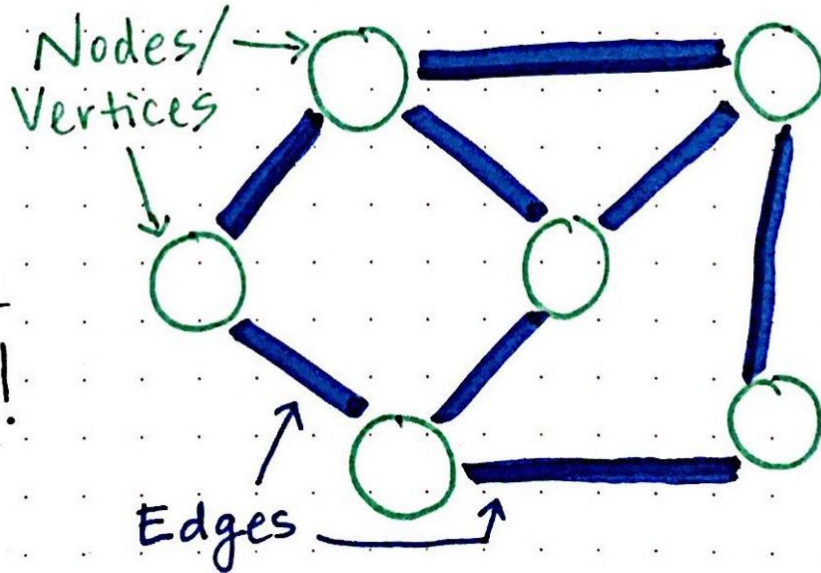
*In mathematics, graphs are a way to formally represent a network, which is basically just a collection of objects that are all interconnected.*

# Applied to MiMO

- The eigenvalues are the ones that characterize the MIMO channel capacity, while the eigenvector does not affect it (for a single-user point-point MIMO channel).
- The capacity-achieving transmission strategy is to send one data signal in each of the eigendirections. Hence, the eigenvalues tell us how strong these parallel channels are.

# Gentle Introduction

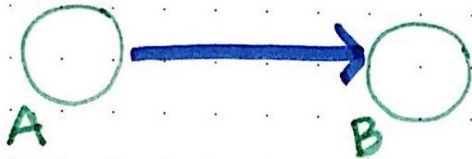
Edges can  
connect nodes  
in any possible  
way! No rules!



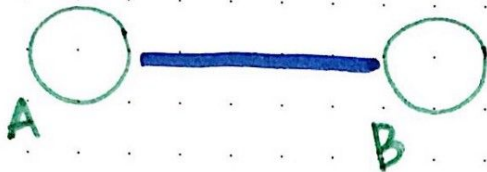


# Edge types

## Different types of edges in graphs

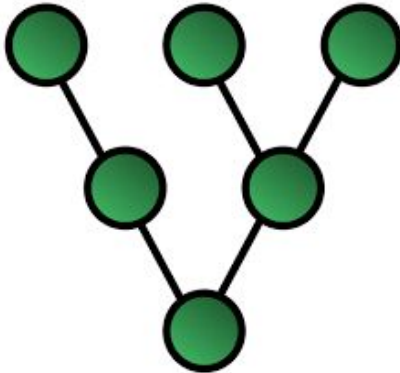
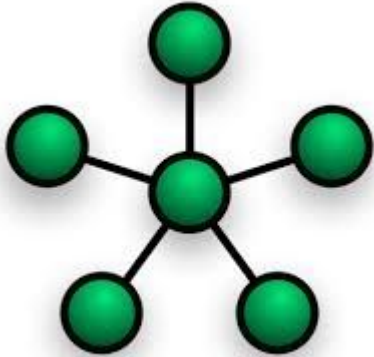
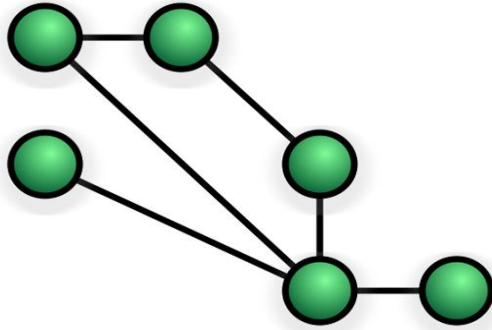
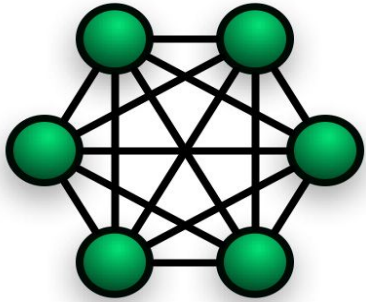


**directed edge:** there is only a path from A, the origin, to B, the destination

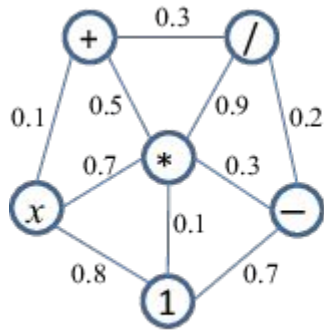


**undirected edge:** the path between A and B is bidirectional, meaning origin & destination are not fixed.

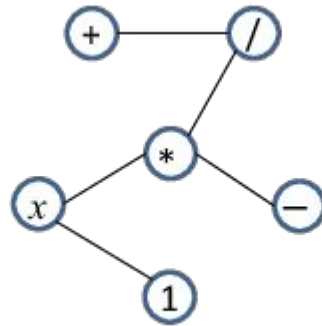
# Graph examples



# Graph->Tree->Distance Matrix



Weighted graph



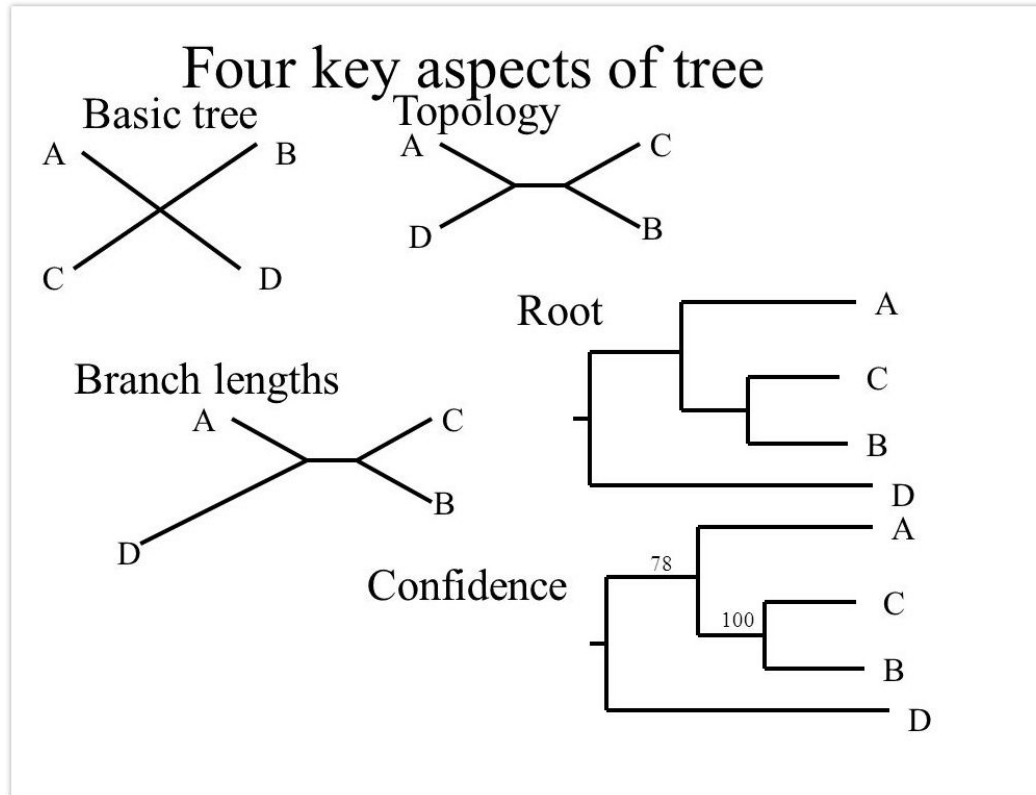
Minimum spanning tree



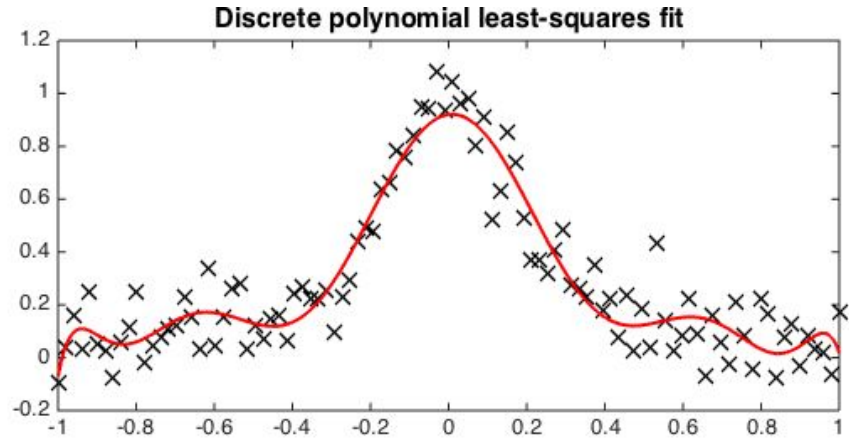
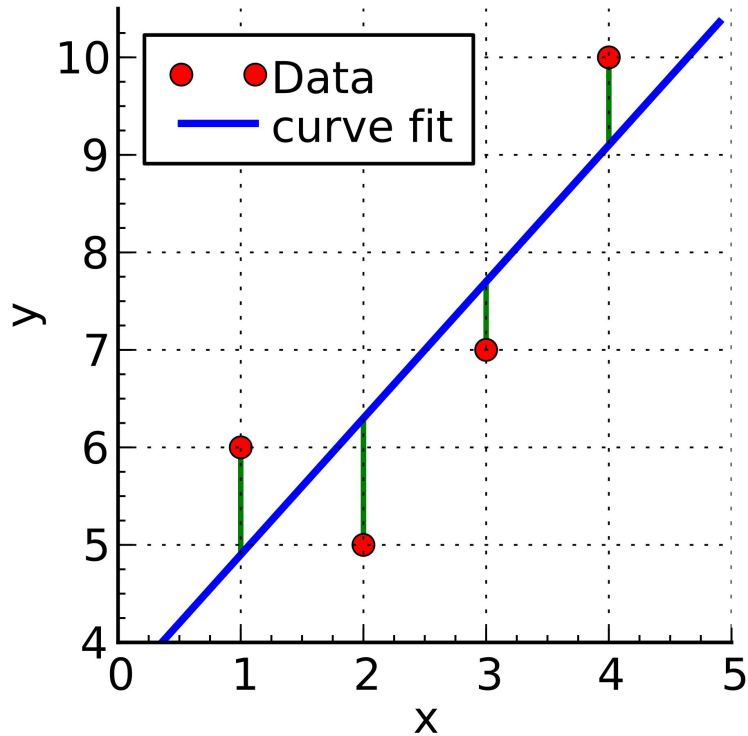
	+	-	*	/	x	1
+						
-	3					
*	2	1				
/	1	2	1			
x	3	2	1	2		
1	4	3	2	3	1	

Distance matrix

# Tree examples



# Least Squares Fitting



Building from available parts

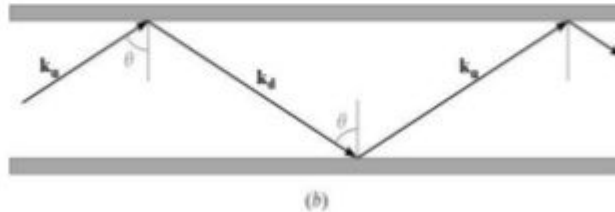
# Wave guides



## Waveguide

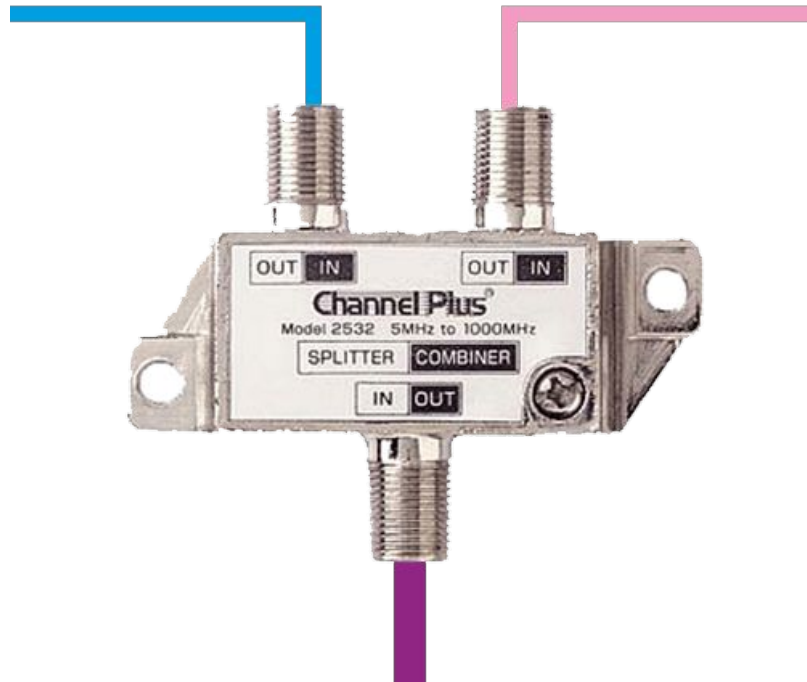
- A waveguide is a structure that guides waves, such as [electromagnetic waves](#) or [sound waves](#). They enable a signal to propagate with minimal loss of energy by restricting expansion to one dimension or two.
- Zigzag reflection, waveguide mode, cutoff frequency

$$|\mathbf{k}_u| = |\mathbf{k}_d| = k = \omega\sqrt{\mu\epsilon}$$

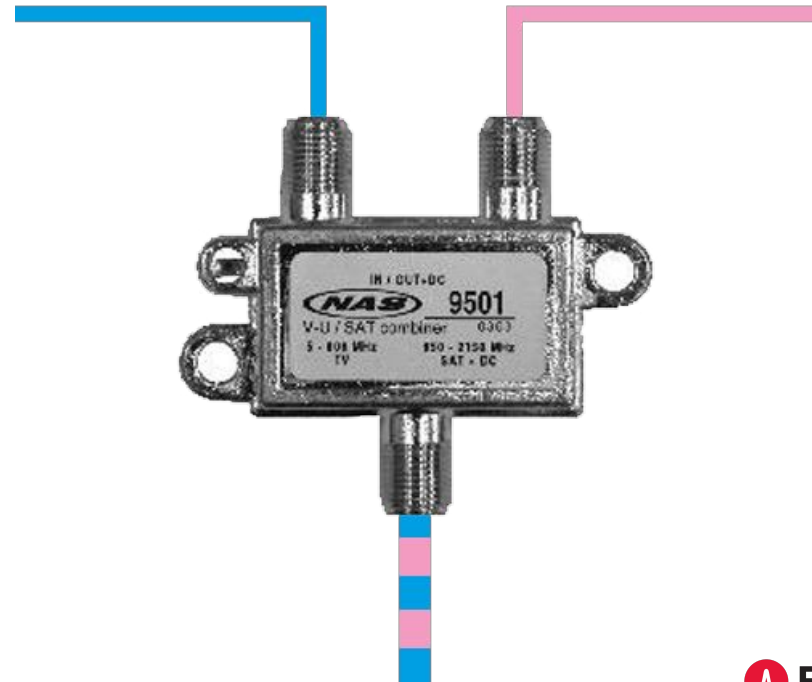


Fixed parts: e.g. Splitter/Combiner, Diplexers

## COMBINER

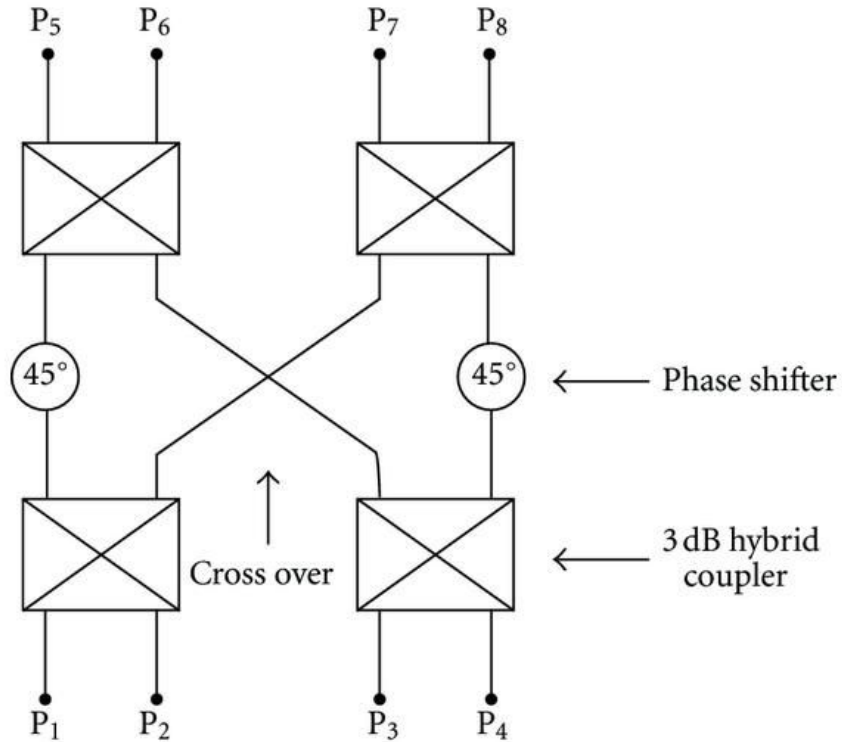


## DIPLEXER





# Butler Matrix

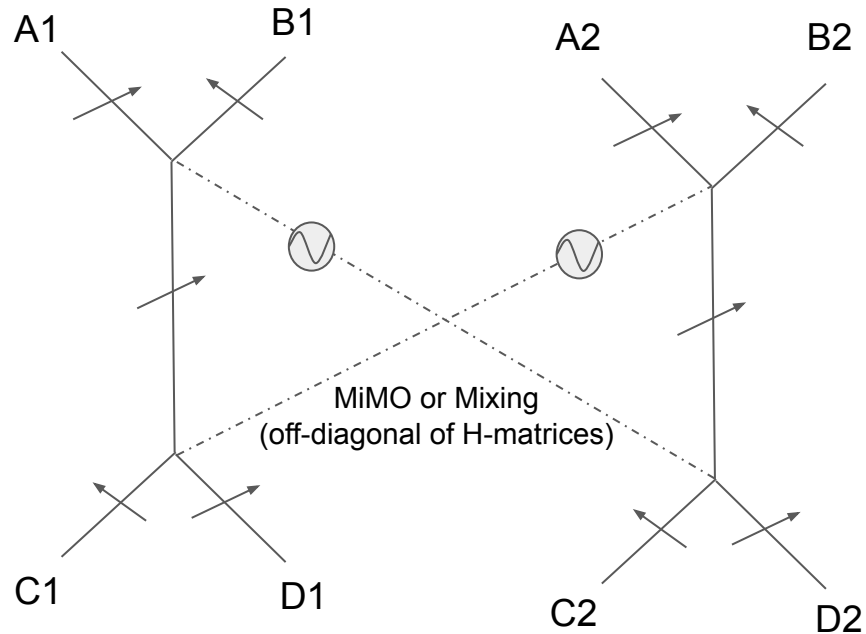


# Programmable parts: attenuators and phase shifters



# Generalized WiFi Graphs

# Four Node, 5-Branch Tree with 2x2 MIMO

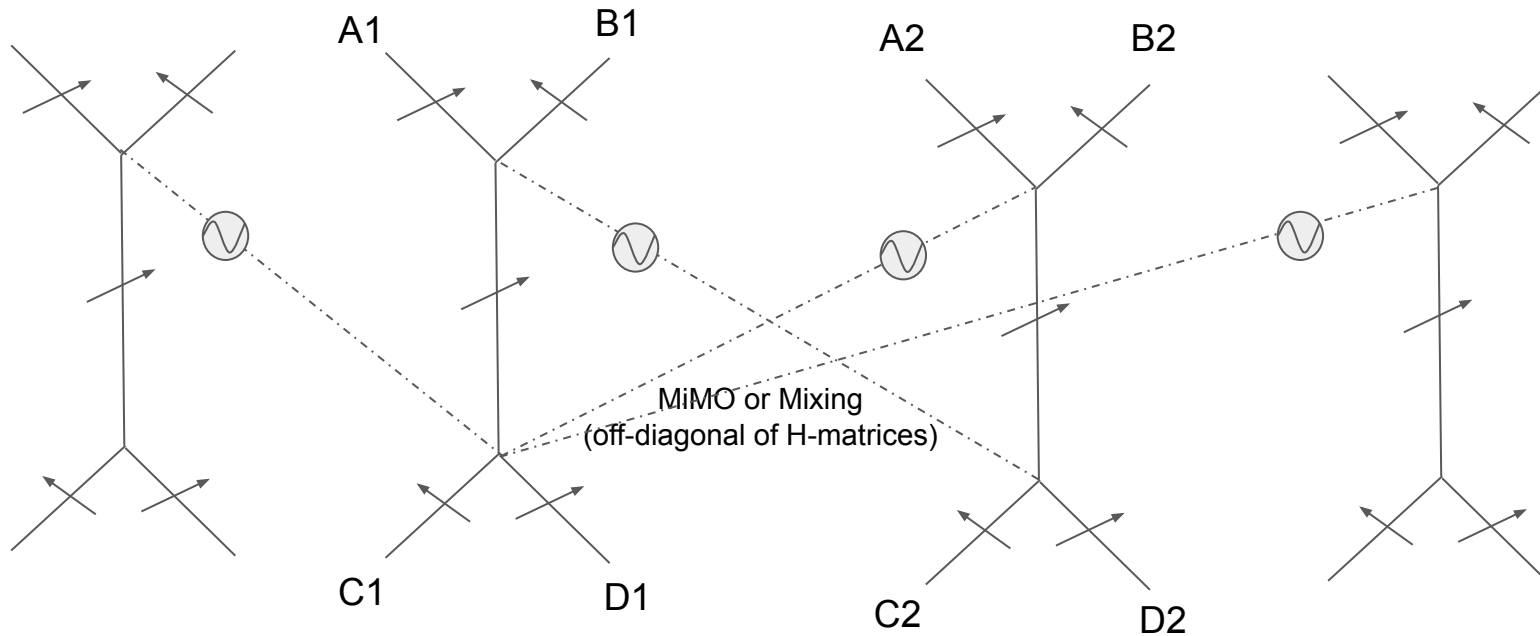


----- Mimo path w/digital programmable phase shifter

——— Path w/digital programmable attenuator

**\*Variable notch filter for OFDMA in discussion**

# Four Node, 5-Branch Tree with 4x4 MIMO



..... Mimo path w/digital programmable phase shifter

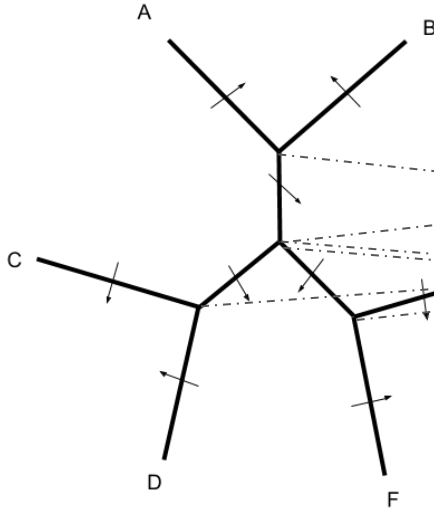
— Path w/digital programmable attenuator

**\*Variable notch filter for OFDMA in discussion**

# 6 Node, 3 BSS topology (3 BSS, 6 nodes, 2x2 MIMO)

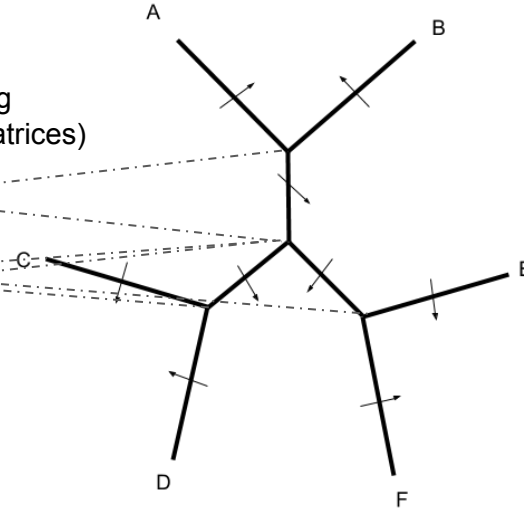
Board / Layer 1 (A-F) Ant 1

Six Nodes (three stars)



Board / Layer 2 (A' - F') Ant 2

Six Nodes (three stars)



MiMO or Mixing  
(off-diagonal of H-matrices)



----- Mimo path w/digital programmable phase shifter

——— Path w/digital programmable attenuator

**\*Variable notch filter for OFDMA in discussion**

# Python Code

# Minimizing Total Error (Partial diff eq)

```
#
# S1                S3
#  \                /
#  a \      b      /c
#      \_____/
#      /                \
#  d /                \ e
#  /                  \
# S2                  S4
#
# D12 = a + d
# D13 = a + b + c
# D14 = a + b + e
# D23 = d + b + c
# D24 = d + b + e
# D34 = c + e
#
# Q = (D12 - a - d)**2 + (D13 - a - b - c)**2 + (D14 - a - b - e)**2 - (D23 - d - b - c)**2 + (D24 - d - b - e)**2 + (D34 - c - e)**2
#
# Substitute the desired RSSI (or distance) into D(i,j) then
# Find the minimum for Q by taking the partial derivatives respect to a,b,c,d and e
# and then solve the five simultaneous equations
```



# Python Code

$$Q = (D12 - a - d)**2 + (D13 - a - b - c)**2 + (D14 - a - b - e)**2 - (D23 - d - b - c)**2 + (D24 - d - b - e)**2 + (D34 - c - e)**2$$

```
@property
```

```
def attnsettings(self):
```

```
    # Create the expression for the tree, i.e. the Summation of the squared errors
```

```
    q = ((Float(self.distances[0]) - self.a - self.d) ** 2) + ((Float(self.distances[1]) - self.a - self.b - self.c) ** 2) + ((Float(self.distances[2]) - self.a - self.b - self.e) ** 2) + ((Float(self.distances[3]) - self.d - self.b - self.c) ** 2) + ((Float(self.distances[4]) - self.d - self.b - self.e) ** 2) + ((Float(self.distances[5]) - self.c - self.e) ** 2)
```

```
    # perform the five partial differentiations and solve the five equations to minimize Q
```

```
    solution = linsolve([diff(q, self.a), diff(q, self.b), diff(q, self.c), diff(q, self.d), diff(q, self.e)], self.a, self.b, self.c, self.d, self.e)
```

```
    (a0, b0, c0, d0, e0) = next(iter(solution))
```

```
    error = q.subs([(self.a, a0), (self.b, b0), (self.c, c0), (self.d, d0), (self.e, e0)])
```

```
    # print("{}".format(solve([diff(q, self.a), diff(q, self.b), diff(q, self.c), diff(q, self.d), diff(q, self.e)])))
```

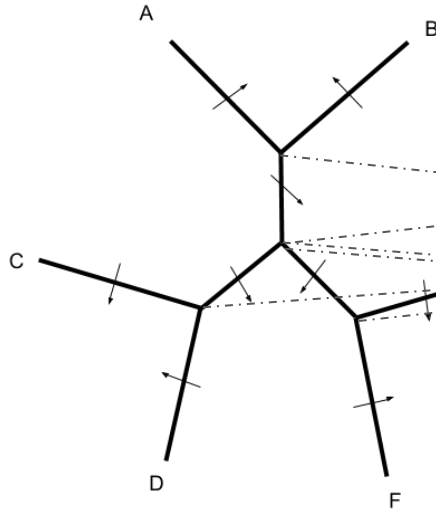
```
    print('Error: {:.2f}'.format(float(sqrt(error))))
```

```
    return solution
```

# Common/Generalized WiFi topology (3 BSS, 6 nodes, 2x2 MIMO)

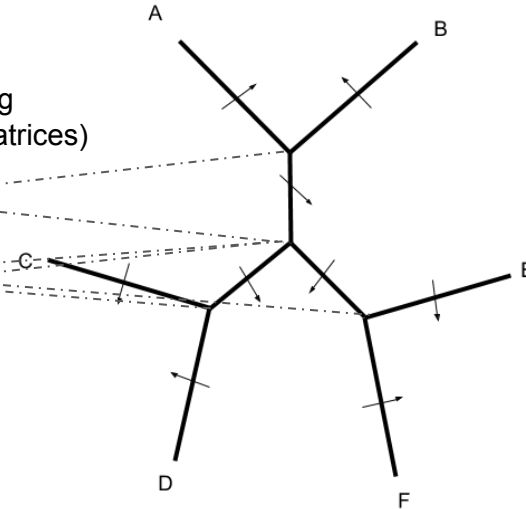
Board / Layer 1 (A-F) Ant 1

Six Nodes (three stars)

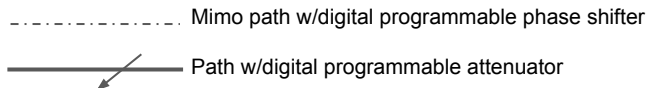


Board / Layer 2 (A' - F') Ant 2

Six Nodes (three stars)



MIMO or Mixing  
(off-diagonal of H-matrices)



**\*Variable notch filter for OFDMA in discussion**

# 6 Nodes, (11 degrees vs 22)

```
#          S3  S4
#          \  /
#          f\ /g
#          \ /
# S1       *
#          \  /
# d\       /b
#          \ a /
#          *----*
#          /   \
#          /    \c
# e/          \
# /           *
# S2          /\
#           h/ \i
#           /  \
#           S5  S6
#
```

```
# 0) D12 = d + e
# 1) D13 = d + a + b + f
# 2) D14 = d + a + b + g
# 3) D15 = d + a + c + h
# 4) D16 = d + a + c + i
# 5) D23 = e + a + b + f
# 6) D24 = e + a + b + g
# 7) D25 = e + a + c + h
# 8) D26 = e + a + c + i
# 9) D34 = f + g
# 10) D35 = f + b + c + h
# 11) D36 = f + b + c + i
# 12) D45 = g + b + c + h
# 13) D46 = g + b + c + i
# 14) D56 = h + i
```

# Python Code

$$Q = (D12 - d - e)**2 + (D13 - d - a - c)**2 + (D14 - a - b - e)**2 - (D23 - d - b - c)**2 + (D24 - d - b - e)**2 + (D34 - c - e)**2$$

```
@property
def attnsettings(self):
    # Create the expression for the tree, i.e. the Summation of the squared errors
    q = ((Float(self.distances[0]) - self.d - self.e) ** 2) + ((Float(self.distances[1]) - self.d - self.a - self.b - self.f) ** 2) + ((Float(self.distances[2]) - self.d - self.a - self.b - self.g) ** 2) +
    ((Float(self.distances[3]) - self.d - self.a - self.c - self.h) ** 2) + ((Float(self.distances[4]) - self.d - self.a - self.c - self.i) ** 2) + ((Float(self.distances[5]) - self.e - self.a - self.b - self.f) ** 2) +
    ((Float(self.distances[6]) - self.e - self.a - self.b - self.g) ** 2) + ((Float(self.distances[7]) - self.e - self.a - self.c - self.h) ** 2) + ((Float(self.distances[8]) - self.e - self.a - self.c - self.i) ** 2) +
    ((Float(self.distances[9]) - self.f - self.g) ** 2) + ((Float(self.distances[10]) - self.f - self.b - self.c - self.h) ** 2) + ((Float(self.distances[11]) - self.f - self.b - self.c - self.i) ** 2) +
    ((Float(self.distances[12]) - self.g - self.b - self.c - self.h) ** 2) + ((Float(self.distances[13]) - self.g - self.b - self.c - self.i) ** 2) + ((Float(self.distances[14]) - self.h - self.i) ** 2)
    # perform the nine partial differentiations and solve the nine equations to minimize Q
    solution = linsolve((diff(q, self.a), diff(q, self.b), diff(q, self.c), diff(q, self.d), diff(q, self.e), diff(q, self.f), diff(q, self.g), diff(q, self.h), diff(q, self.i)), self.a, self.b, self.c, self.d, self.e, self.f, self.g,
self.h, self.i)
    (a0, b0, c0, d0, e0, f0, g0, h0, i0) = next(iter(solution))
    error = q.subs([(self.a,a0),(self.b,b0),(self.c,c0),(self.d,d0),(self.e,e0), (self.f,f0),(self.g,g0),(self.h,h0), (self.i,i0)])
    print('Error:{:0.2f}'.format(float(sqrt(error))))
    return solution
```

# Take aways

- [Seminal MIMO paper](#)
- David Reed's [myth of interference](#)
- [Least Squares Five Branch Tree link](#)